Math 250 Briggs 4.9

Antiderivatives and Indefinite Integration

Objectives

1) Use basic integration rules to find general antiderivatives (with $+C$)

2) Write the general solution of a differential equation. (with $+C$)

3) Use indefinite integral notation for antiderivatives

4) Find a particular solution of a differential equation (find value of $+C$)

5) vertical motion

$$s''(t) = -32 \text{ ft/sec}^2 \quad \text{or} \quad -9.8 \text{ m/sec}^2$$

$$s'(t) = \int s''(t)\, dt$$

$$s(t) = \int s'(t)\, dt$$

What is the function whose derivative is $f(x)$?
Let's call it $F(x)$.
So $F'(x) = f(x)$
$F''(x) = f'(x)$
$F'''(x) = f''(x)$ etc.

$F(x)$ is called <u>an antiderivative of $f(x)$</u> because $F'(x) = f(x)$.

① <u>Example</u> Find an antiderivative of $f(x) = 3x^4$. Check by differentiatin[g]

$F(x)$ must have an exponent one greater: $4+1=5$

$F(x)$ must have a constant that multiplies with that exp 5 to get coefficient 3: $3 \cdot \frac{1}{5} = \frac{3}{5}$

$F(x) = \frac{3}{5}x^5$

check: $F'(x) = \frac{3}{5} \cdot 5 \cdot x^4 = 3x^4$ ✓

$\boxed{F(x) = \frac{3}{5}x^5}$ is an antiderivative of $f(x) = 3x^4$

② <u>Example</u> Could $H(x) = \frac{3}{5}x^5 + 4$ also be an antiderivative of $f(x) = 3x^4$? Check by differentiating:

$H'(x) = \frac{3}{5} \cdot 5x^4 + 0 = 3x^4 = f(x)$.

$\boxed{\text{yes}}$, $H(x)$ is also an antiderivative of $f(x) = 3x^4$

③ <u>Example</u> How many antiderivatives of $f(x) = 3x^4$ exist?

$\boxed{\text{Infinitely many.}}$

We can add any constant to $\frac{3}{5}x^4$ to get an antiderivative.

$G(x) = \frac{3}{5}x^4 + C$ is called <u>the general antiderivative</u> of $f(x) = 3x^4$ because tells us all of the possibilities. (proof p. 248)

C is called <u>the constant of integration</u> and

* must always be included in the answer for indefinite integrals *

So far, we have used derivative notation, which shows that we differentiate the answer: $F'(x) = f(x)$.

We want notation that shows what we do to $f(x)$:

Let's use $y' = f(x)$ $\quad \left\{ \begin{array}{l} \text{so } y' = F'(x) \\ \text{which means } y = F(x) + C \end{array} \right\}$

which is $\frac{dy}{dx} = f(x)$

which is $dy = f(x)\,dx$ (differential form)

General Rule for polynomials:

If $f(x) = ax^n$

$F(x) = a \cdot \frac{1}{n+1} \cdot x^{n+1} + C$

Review: List our derivatives of trig functions:

$\frac{d}{dx} \sin x = \cos x$

$\frac{d}{dx} \cos x = -\sin x$

$\frac{d}{dx} \tan x = \sec^2 x$

$\frac{d}{dx} \cot x = -\csc^2 x$

$\frac{d}{dx} \sec x = \sec x \tan x$

$\frac{d}{dx} \csc x = -\csc x \cot x.$

Functions	Antiderivatives
if $f(x) = \cos x$	then $F(x) = \sin x + C$
if $f(x) = -\sin x$	then $F(x) = \cos x + C$
or $f(x) = \sin x$	then $F(x) = -\cos x + C$
if $f(x) = \sec^2 x$	then $F(x) = \tan x + C$
if $f(x) = -\csc^2 x$	then $F(x) = \cot x + C$
or $f(x) = \csc^2 x$	then $F(x) = -\cot x + C$
if $f(x) = \sec x \tan x$	then $F(x) = \sec x + C$
if $f(x) = -\csc x \cot x$	then $F(x) = \csc x + C$
or $f(x) = \csc x \cot x$	then $F(x) = -\csc x + C$

M250

Integral notation

$$\int f(x)\,dx = F(x) + C$$

The symbol $\int$ is called an indefinite integral

so $\int f(x)\,dx$ is the "indefinite integral of $f(x)$"

Instructions may say

- Find the indefinite integral $\int f(x)\,dx$
- Integrate $f(x)$.
- Anti-differentiate $f(x)$
- Solve the differential equation $y' = f(x)$
- Find all antiderivatives of $f(x)$.

These all mean

- Find $F(x) + C$ so that $F'(x) = f(x)$.

$$\int f(x)\,dx = F(x) + c = \int dy = y(x) + C$$

and $$\int f'(x)\,dx = F'(x) + C = y'(x) + C$$

$$\int f''(x)\,dx = F''(x) + C = y''(x) + C$$

$\Rightarrow$ To integrate once is to remove one derivative.

If we use indefinite integral notation

$$\int ax^n \, dx = \frac{a}{n+1} x^{n+1} + C \qquad \text{general power rule}$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C.$$

Notice what's missing!

We do <u>not</u> (yet) have these antiderivatives:

$$\int \tan x \, dx$$

$$\int \cot x \, dx$$

$$\int \sec x \, dx$$

$$\int \csc x \, dx$$

M250

Properties of Indefinite Integrals

1) $\int c \cdot f(x)\,dx = c\int f(x)\,dx$

constant multiple rule

2) $\int f(x)+g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$

sum rule

3) $\int f(x) - g(x)\,dx = \int f(x)\,dx - \int g(x)\,dx$

difference rule

Important Note: Since integration the reverse process of differentiation, it, like derivatives, cannot be split as products or quotients

No: $\int \frac{f(x)}{g(x)}\,dx \neq \frac{\int f(x)\,dx}{\int g(x)\,dx}$

No: $\int f(x)\cdot g(x)\,dx \neq \int f(x)\,dx \cdot \int g(x)\,dx$

Find general antiderivatives; check by differentiating

4) Example $f(x) = 8$

$\boxed{F(x) = 8x + C}$ check $F'(x) = 8 = f(x)$ ✓

5) Example $f(x) = x^{1/2}$

$\boxed{F(x) = \frac{2}{3}x^{3/2} + C}$ check $F'(x) = \frac{2}{3} \cdot \frac{3}{2} \cdot x^{3/2 - 1} + 0 = x^{1/2}$ ✓

6) Example $f(t) = (2t^2 - 1)^2$

$f(t) = 4t^4 - 4t^2 + 1$

$\boxed{F(t) = \frac{4}{5}t^5 - \frac{4}{3}t^3 + t + C}$ check $F'(x) = \frac{4}{5} \cdot 5t^4 - \frac{4}{3} \cdot 3t^2 + 1$ ✓

Find indefinite integrals. Check by differentiating.

⑦ Example $\int \cos x \, dx = \boxed{\sin x + C}$

check $\frac{d}{dx}(\sin x + C) = \cos x + 0$ ✓

⑱ $\int \sec y (\tan y - \sec y) \, dy$

$= \int (\sec y \cdot \tan y - \sec^2 y) \, dy$

$= \boxed{\sec y - \tan y + C}$

check $\frac{d}{dy}(\sec y - \tan y + c)$

$= \sec y \tan y - \sec^2 y + 0$ ✓

same as #5

⑨ $\int x^{1/2} \, dx$

$= \frac{2}{3} x^{1/2+1} + C = \boxed{\frac{2}{3} x^{3/2} + C}$

check $\frac{d}{dx}\left(\frac{2}{3} x^{3/2} + c\right) = x^{1/2} + 0$ ✓

⑩ $\int \frac{x^2 + 2x - 1}{\sqrt{x}} \, dx$

$= \int \left(\frac{x^2}{x^{1/2}} + \frac{2x}{x^{1/2}} - \frac{1}{x^{1/2}} \right) dx$

$= \int \left(x^{3/2} + 2x^{1/2} - x^{-1/2} \right) dx$

$= \frac{2}{5} x^{5/2} + \frac{4}{3} x^{3/2} - 2x^{1/2} + C$

check

$\frac{d}{dx}\left(\frac{2}{5} x^{5/2} + \frac{4}{3} x^{3/2} - 2x^{1/2} + c \right)$

$= x^{3/2} + 2x^{1/2} - x + 0$ ✓

* Factor out least powers and common denominator for fully-simplied final answer *

$$= \frac{2}{15} x^{1/2} \left[\frac{\frac{2}{5} x^{5/2}}{\frac{2}{15} x^{1/2}} + \frac{\frac{4}{3} x^{3/2}}{\frac{2}{15} x^{1/2}} - \frac{2x^{1/2}}{\frac{2}{15} x^{1/2}} \right] + C$$

cont →

divide coefficients and subtract exponents

$$\frac{2}{5} \div \frac{2}{15} = \frac{2}{5} \cdot \frac{15}{2} = 3$$

$$\frac{x^{5/2}}{x^{1/2}} = x^{5/2 - 1/2} = x^{4/2} = x^2$$

$$= \boxed{\frac{2}{15} x^{1/2} \left[3x^2 + 10x - 15 \right] + C}$$

(11) Solve the differential equation

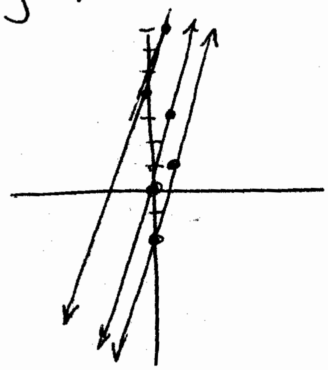
$y' = 3$.

$$y = \int y' \, dx = \int 3 \, dx = 3x + C$$

$\boxed{y(x) = 3x + C}$ ← This is called the <u>general solution</u> of this differential equation (DE) because this is a family of functions, any one of which is a solution of $y' = 3$.

(12) Sketch graphs of $y(x) = 3x + C$ when $C = -2, 0$ and 4.

$y(x) = 3x - 2$ $\quad$ $y(x) = 3x + 0$ $\quad$ $y(x) = 3x + 4$



Other types of problems that appear in this section:

1) Solve the differential equation
$y' = f(x)$ when $y(0) = c$
This is called an initial value problem (IVP) because information is given so we can find c.

2) Solve the differential equation
$y'' = f(x)$ when $y'(0) = c_1$ and $y''(0) = c_2$
This is a 2nd-order DE because we integrate twice

M250

(13) Solve the initial value problem $\frac{dy}{dx} = 3$ when $y(0) = 4$.

GOAL: Find a function $y(x)$ so that $\frac{dy}{dx} = 3$

and $y(0) = 4$.

step 1: Write the differential equation in differential form.

$$dy = 3\,dx$$

step 2: Integrate both sides of the equation according to the differential.

$$\int dy = \int 3\,dx$$

$$y = 3x + C$$ ← This is the general solution to the DE.

step 3: Find c so $y(0) = 4$.

$$y(0) = 3(0) + C = 4$$
$$C = 4$$

step 4: Rewrite final answer

$$\boxed{y(x) = 3x + 4}$$

check: $\frac{dy}{dx} = 3$ ✓

(15) Neglecting air resistance, the motion of an object moving vertically near Earth's surface is determined by the acceleration due to gravity, $a(t) = g$, a constant.

$g = -9.8\ m/s^2$ if metric units
$g = -32\ ft/s^2$ if English units
$\}$ * memorize these values!

Suppose a stone is thrown vertically upward at $t=0$ with a velocity of 40 m/s from the edge of a cliff 100 m above a river.

a) Find the velocity $v(t)$ of the object for $t \geq 0$.
b) Find the position $s(t)$ of the object, for $t \geq 0$.
c) Find the maximum height of the object above the river.
d) With what speed does the object strike the water?

a) acceleration $a(t) = g = -9.8\ m/s^2$

velocity $v(t) = \int a(t)\,dt$

$$= \int -9.8\,dt$$

$$= -9.8t + C_1$$

use initial velocity to find C_1

$$v(0) = -9.8(0) + C_1 = 40\ m/s$$

$$0 + C_1 = 40$$

$$C_1 = 40$$

$$\boxed{v(t) = -9.8t + 40}$$

b) position $= \int v(t)\,dt$

$$s(t) = \int -9.8t + 40\,dt$$

$$= \frac{-9.8t^2}{2} + 40t + C_2$$

$$s(t) = -4.9t^2 + 40t + C_2$$

use initial position to find C_2

$$s(0) = -4.9(0)^2 + 40(0) + C_2 = 100$$

$s(0) = 0 + 0 + C_2 = 100$

$C_2 = 100$

so $\boxed{s(t) = -4.9t^2 + 40t + 100}$

c) Find the maximum height.

Algebra method:
vertex formula

$t = \frac{-b}{2a}$

$t = \frac{-40}{2(-4.9)}$

$t = \frac{-40}{-9.8}$

$t = +\frac{200}{49}$ s ≈ 4.1 sec

This is another way to derive the vertex formula!

Calculus method
Find relative extrema of $s(t)$.

$s'(t) = v(t) = 0$

$-9.8t + 40 = 0$

$t = \frac{-40}{-9.8}$

$t = \frac{200}{49}$ sec

max height $= s\left(\frac{200}{49}\right)$

$$s\left(\frac{200}{49}\right) = -4.9\left(\frac{200}{49}\right)^2 + 40\left(\frac{200}{49}\right) + 100$$

$$= \boxed{\frac{8900}{49} \text{ m}} \approx 181.6 \text{ m.}$$

d) With what speed ($|v(t)|$) does the stone strike the river?

strike the river $\Rightarrow$ $s(t) = 0$
solve for t^*

$\Rightarrow$ speed $|v(t^*)|$

$$s(t) = -4.9t^2 + 40t + 100 = 0$$

QF or CTS

$$t^* = \frac{-40 \pm \sqrt{40^2 - 4(-4.9)(100)}}{2(-4.9)}$$

$$t^* = \frac{-40 \pm \sqrt{3560}}{-9.8}$$

3560

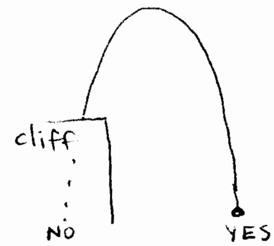
10 356

2 5 4 89

$2\sqrt{890}$

$$t^* = \frac{40}{9.8} \pm \frac{2\sqrt{890}}{9.8}$$

$$t^* = \frac{200}{49} \pm \frac{10\sqrt{890}}{49}$$



$$t^* = \frac{200}{49} + \frac{10\sqrt{890}}{49} \approx 10.2 \text{ sec}$$

$$v(t^*) = -9.8\left(\frac{40}{9.8} + \frac{2\sqrt{890}}{9.8}\right) + 40$$

$$= -40 \quad 2\sqrt{890} + 40$$

$$= -2\sqrt{890} \approx -59.7 \text{ m/sec}$$

speed = |velocity|

$$\text{speed} = \boxed{2\sqrt{890} \text{ m/s}} \approx +59.7 \text{ m/s}$$

Math 250

(14) Solve the differential equation $f''(x) = x^2$ when $f'(0) = 8$ and $f(0) = 4$.

Step 1: Integrate first time to find f'

$$\int f''(x)\,dx = \int x^2\,dx$$

$$f'(x) = \frac{1}{3}x^3 + C_1$$

Step 2: Substitute $f'(0) = 8$ to find value of C_1.

$$f'(0) = \frac{1}{3}(0)^3 + C_1 = 8$$

$$C_1 = 8$$

$$f'(x) = \frac{1}{3}x^3 + 8$$

Step 3: Integrate second time to find f

$$\int f'(x)\,dx = \int \frac{1}{3}x^3 + 8\,dx$$

$$f(x) = \frac{1}{12}x^4 + 8x + C_2$$

Step 4: Substitute $f(0) = 4$ to find C_2

$$f(0) = \frac{1}{12}(0)^4 + 8(0) + C_2 = 4$$

$$C_2 = 4$$

$$\boxed{f(x) = \frac{1}{12}x^4 + 8x + 4}$$

Rectilinear Motion

A particle or object moves along a line

$s(t)$ gives its <u>position</u> at time t.
(or $x(t)$)

$s(t) > 0$ → to right of origin
$s(t) < 0$ → to left of origin
$s(t) = 0$ at the origin

$v(t) = s'(t)$ gives its <u>velocity</u> at time t

$v(t) > 0$ → moving right
$v(t) < 0$ → moving left
$v(t) = 0$ not moving = position constant

$a(t) = v'(t) = s''(t)$ gives its <u>acceleration</u> at time t

$a(t) > 0$ → velocity increasing
$a(t) < 0$ → velocity decreasing
$a(t) = 0$ → velocity constant

If we are given acceleration: $a(t)$ then we can find velocity and position by anti-differentiating:

$$\boxed{\begin{aligned} v(t) &= \int a(t)\,dt \\ s(t) &= \int v(t)\,dt \end{aligned}}$$

Note: These are <u>indefinite</u> integrals, so these antiderivatives have a $+C$ constant of integration.

Constant acceleration

Given: $a(t) = a$ $\quad$ a is a constant

Then: $v(t) = \int a(t)\,dt$

$= \int a\,dt$

$v(t) = at + C_1$ $\quad \Rightarrow \quad v(0) = a \cdot 0 + C_1$

$v(0) = C_1$

so C_1 is always the initial velocity. Call it v_0.

Velocity function for constant acceleration

$$\boxed{v(t) = at + v_0}$$

$s(t) = \int v(t)\,dt$

$= \int at + v_0\,dt$ $\quad$ recall a, v_0 are constants.

$s(t) = \frac{1}{2}at^2 + v_0 t + C_2$

$\Rightarrow \quad s(0) = \frac{1}{2}a(0)^2 + v_0(0) + C_2$

$s(0) = C_2$

C_2 is always the initial position. call it s_0.

Briggs ~~[illegible]~~ 4.9

Needed Th 10/19.

Position function for constant acceleration

$$\boxed{s(t) = \frac{1}{2}at^2 + v_0 t + s_0}$$

Free-fall model = Constant acceleration
+ vertical motion: $s(t)$ up vs. down.
$(+)$ $(-)$
+ acceleration due to gravity
is a specific value which is fixed but depends on units used.

$$a(t) = -g$$

Negative ⇒ gravity pulls down.

g is constant

* $g = 32\ ft/s^2$ in English units
$g = 9.8\ m/s^2$ in Metric units

$$v(t) = \int a(t)\,dt$$
$$= \int -g\,dt$$
$$\boxed{v(t) = -gt + v_0}$$

velocity function rectilinear motion due to gravity (up/down)

$$s(t) = \int v(t)\,dt$$
$$= \int -gt + v_0\,dt$$
$$\boxed{s(t) = -\tfrac{1}{2}gt^2 + v_0t + s_0}$$

position function rectilinear motion due to gravity (up/down)

Example

(15) A particle, initially at rest, moves along the x-axis such that its acceleration at time $t>0$ is given by $a(t) = \cos t$. At time $t=0$, its position is $x=3$.

a) Find the velocity and position functions for the particle

b) Find the values of t for which the particle is at rest

a) velocity is both $\begin{cases} v(t) = x'(t) \\ v(t) = \int a(t)\,dt = \int v'(t)\,dt \end{cases}$ ← not useful unless given $x(t)$. ← use this

$$v(t) = \int \cos t \, dt$$

$$v(t) = \sin t + C$$

"initially at rest" $\Rightarrow v(0) = 0$ ↖ { "at rest" = not moving; "initially" = at time 0 }

$$v(0) = \sin(0) + C = 0$$

means $C = 0$.

$$\boxed{v(t) = \sin(t)}$$

position $x(t) = \int v(t)\,dt = \int x'(t)\,dt$

$$x(t) = \int \sin t \, dt$$

$$x(t) = -\cos t + C$$

"at time 0, position is $x=3$" means $x(0)=3$

$$x(0) = -\cos(0) + C = 3$$

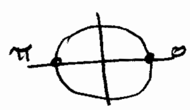
$$-1 + C = 3$$

$$C = 4$$

$$\boxed{x(t) = -\cos(t) + 4} \text{ or } \boxed{x(t) = 4 - \cos(t)}$$

b) "particle at rest" means $v(t) = 0$.

$$v(t) = \sin(t) = 0$$



$$\boxed{t = k\pi \quad k \in \mathbb{W}}$$

↑ $t>0$, so no negatives

$\mathbb{W}$ means whole numbers: $\{0, 1, 2, 3 \ldots\}$

<u>Note</u>: $\cos(t)$ is NOT constant

So this is <u>NOT</u> constant acceleration

Example

(16) A car traveling at 45 mph is brought to a stop, at constant deceleration, 132 feet from where the brakes are applied.

a) How far has the car moved when its speed has been reduced to 30 mph?

b) " " " " " " " " " " " " 15 mph?

c) Draw the real number line from 0 to 132 and plot the points found in a) and b). What can you conclude?

$a(t) = a_0 < 0$ "constant deceleration".

$v(t) = \int a(t)\,dt = \int a_0\,dt = a_0 t + v_0$

$v(0) = 45 = a_0 \cdot (0) + v_0$

$\Rightarrow V_0 = 45.$

$\Rightarrow a(t) = a_0$

$v(t) = a_0 t + 45$ ← (a_0 still unknown)

132 feet vs 45 miles per hour — units mismatch must repaired.

Convert 132 ft to miles: $\frac{132\text{ ft}}{1} \cdot \frac{1\text{ mi}}{5280} = 0.025\text{ mi} = \frac{1}{40}\text{ mi}.$

$x(t) = \int v(t)\,dt = \int (a_0 t + 45)\,dt = \frac{1}{2}a_0 t^2 + 45t + x_0$

$x(0) = 0 = \frac{1}{2}a_0(0)^2 + 45(0) + x_0$

$\Rightarrow x_0 = 0$

$\Rightarrow x(t) = \frac{1}{2}a_0 t^2 + 45t$ ← (a_0 still unknown)

Find time the car stops: (call it $\overset{*}{t}$)

$$\begin{cases} \frac{1}{40} = \frac{1}{2}a_0 \overset{*}{t}^2 + 45\overset{*}{t} & \leftarrow \text{using } x(\overset{*}{t}) = \frac{1}{40}\text{ mi} \\ 0 = a_0\overset{*}{t} + 45 & \leftarrow \text{using } v(\overset{*}{t}) = 0\text{ mph} \end{cases}$$

$-45 = a_0\overset{*}{t}$

time car stops: $-\frac{45}{a_0} = \overset{*}{t}$

{Subst / Evaluate $x(\overset{*}{t}) = \frac{1}{40}$:

$\frac{1}{40} = \frac{1}{2}a_0\left(\frac{-45}{a_0}\right)^2 + 45\left(\frac{-45}{a_0}\right)$

Get eqn w/ only a_0!

$\frac{1}{40} = \frac{1}{2}a_0\left(\frac{2025}{a_0^2}\right) - \frac{2025}{a_0}$

Solve for a_0:

$\frac{1}{40} = \frac{2025}{2}\cdot\frac{1}{a_0} - 2025 \cdot \frac{1}{a_0}$

$\frac{1}{40} = -\frac{2025}{2}\cdot\frac{1}{a_0}$

$a_0 = -\frac{2025}{2}\cdot 40 = -40500\text{ mi}/\text{hr}^2.$

We now know all 3 functions ☺

$$\Rightarrow \begin{cases} a(t) = -40500 \\ v(t) = -40500t + 45 \\ x(t) = -20250t^2 + 45t \end{cases}$$

a) $v(t) = 30$

$-40500t + 45 = 30$

$-40500t = -15$

$t = \frac{1}{2700}\text{ hr}.$

$x\left(\frac{1}{2700}\right) = -20250\left(\frac{1}{2700}\right)^2 + 45\left(\frac{1}{2700}\right) = \frac{1}{72}\text{ mi} \cdot \frac{5280\text{ ft}}{1\text{ mi}} = \boxed{73\frac{1}{3}\text{ ft}}$

b) $v(t) = 15$

$-40500t + 45 = 15$

$-40500t = -30$

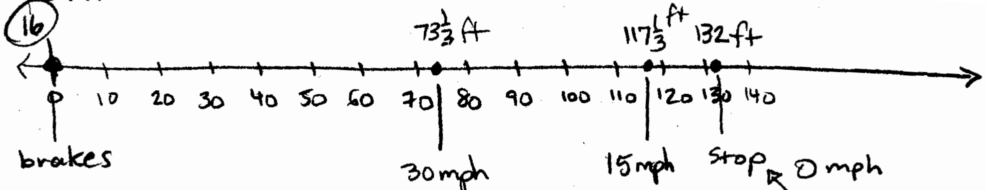
$t = \frac{1}{1350}\text{ hr}$

$x\left(\frac{1}{1350}\right) = -20250\left(\frac{1}{1350}\right)^2 + 45\left(\frac{1}{1350}\right) = \frac{1}{45}\text{ mi} \cdot \frac{5280\text{ ft}}{1\text{ mi}} = \boxed{117\frac{1}{3}\text{ ft}}$

cont next page

cont

c) 16



$\frac{1}{2700}$ hr $= 1.\overline{3}$ sec

$\frac{1}{1350}$ hr $= 2.\overline{6}$ sec

$^{*}t = \frac{-45}{-40500} = \frac{1}{900}$ hr $= 4$ sec

same amount of time needed for each reduction of speed

$$a_0 = \frac{\Delta v}{\Delta t} = \frac{-15 \text{ mph}}{1\frac{1}{3} \text{ sec}}$$ on all intervals

but less distance needed for each reduction in speed.

$$\begin{cases} 73\frac{1}{3} \text{ ft in } 1\frac{1}{3} \text{ sec} \\ 44 \text{ ft in } 1\frac{1}{3} \text{ sec} \\ 14\frac{2}{3} \text{ ft in } 1\frac{1}{3} \text{ sec} \end{cases}$$